AD646855

40

USL Report No. 786

Utilization of the Reciprocity Theorem to Determine The Near Field Air-to-Subsurface Propagation Formulas

PETER R. BANNISTER
E/M Systems Planning Branch
Radio Communication Systems Division

22 November 1966

Distribution of this document is unlimited.

U. S. Navy Underwater Sound Laboratory
Fort Trumbull, New London, Connecticut

F

ABSTRACT

The electric and magnetic field components produced by vertical and horizontal dipoles (both electric and magnetic types) located at or above the surface of a plane, conducting, homogeneous earth are derived for the near field range. The height h of the transmitting antenna is $\geq 0^+$, while the depth z of the receiving antenna is $\leq 0^-$ (air-to-subsurface propagation). Ionospheric effects are neglected.

The derivations are based upon application of the reciprocity theorem to previously derived field components. It is observed that these equations reduce to well-known expressions when the horizontal separation (ρ) between the transmitting and receiving antennas is much greater than h.

ADMINISTRATIVE INFORMATION

The analysis described in this report was performed under USL Problem No. 7-1-901-00-00 as a general theoretical propagation study. The corresponding Navy Subproject and Task No. is SF 106 01 02-7077.

REVIEWED AND APPROVED: 22 November 1966

H. E. Nash
H. E. Nash
Technical Director

R. L. Corkran
R. L. Corkran, Jr., Captain, USN
Commanding Officer and Director

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

Figure

LIST OF TABLES

Table

DEFINITION OF SYMBOLS USED

c	$\cong 3 \times 10^8$ meters/second, velocity of light in free space
dA	infinitesimal dipole area (meters^2)
$d\ell$	infinitesimal dipole length (meters)
E_ρ	horizontal electric field component in the ρ direction (volts/meter)
E_ϕ	horizontal electric field component in the ϕ direction (volts/meter)
E_z	vertical electric field component (volts/meter)
h	height ($h \geq 0^+$) or depth ($h \leq 0^-$) of transmitting antenna with respect to the earth's surface
HED	horizontal electric dipole
HMD	horizontal magnetic dipole
H_ρ	horizontal magnetic field component in the ρ direction (amperes/meter)
H_ϕ	horizontal magnetic field component in the ϕ direction (amperes/meter)
H_z	vertical magnetic field component (amperes/meter)
$\vec{m}$	direction of magnetic dipole axis
0^+	an infinitesimal distance above the earth's surface
0^-	an infinitesimal distance below the earth's surface
R	$(\rho^2 + z^2)^{1/2}$
R_o	$[\rho^2 + (z - h)^2]^{1/2}$
R_1	$[\rho^2 + (z + h)^2]^{1/2}$
R'	$(\rho^2 + h^2)^{1/2}$
u_o	$(\lambda^2 + \gamma_o^2)^{1/2}$ (meters^{-1}) (air)
u_1	$(\lambda^2 + \gamma_1^2)^{1/2}$ (meters^{-1}) (earth)
VED	vertical electric dipole
VMD	vertical magnetic dipole
z	height ($h \geq 0^+$) or depth ($h \leq 0^-$) of receiving antenna with respect to the earth's surface
γ_o	$= (-\epsilon_o \mu_o \omega^2)^{1/2}$ upper half-space (free-space) propagation constant (meters^{-1})
γ_1	$= (i\sigma_1 \mu\omega - \epsilon\mu\omega^2)^{1/2} \cong (i\sigma_1 \mu_o \omega)^{1/2}$, lower half-space (earth) propagation constant (the displacement currents in the earth are assumed to be negligible) (meters^{-1})
δ	$= (2/\omega\mu_o\sigma_1)^{1/2}$, skin depth in lower half space (earth)
ϵ_o	$\cong 10^{-9}/36\pi$ farads/meter, permittivity of free space
λ	dummy integration variable in the basic Sommerfeld integrals
λ_{air}	$= c/f$, free-space wavelength
ρ	$(x^2 + y^2)^{1/2}$ radial distance in a cylindrical coordinate system
σ_1	conductivity of the lower half space (earth) (mhos/meter)
ϕ	$\tan^{-1} y/x$, azimuth angle in a cylindrical coordinate system
$\mu \cong \mu_o$	$= 4\pi \times 10^{-7}$ henries/meter, permeability of free space
ω	$2\pi f$ radians/second, angular frequency

UTILIZATION OF THE RECIPROCITY THEOREM TO DETERMINE THE NEAR FIELD AIR-TO-SUBSURFACE PROPAGATION FORMULAS

INTRODUCTION

Bannister and Bannister and Hart have derived the near field (ρ comparable to λ_{air}) subsurface-to-air electric and magnetic field components produced by vertical and horizontal dipole antennas (both electric and magnetic types).[1] These expressions, which are valid for $|\gamma_1 R| >> 1$ $(R >> \delta)$, $h \leq 0^-$, $z \geq 0^+$, and $R >> |h|$ are listed in Tables 1 and 2. An additional restriction $(|\gamma_0^2 \rho/\gamma_1| << 1)$ is required for the vertically polarized components (E_ρ, E_z, and H_ϕ). This limits the range to small "numerical distances",[2] although $|\gamma_0 \rho|$ may exceed unity (i.e., ρ may be $> \lambda_{air}$).

In the present study, the author will derive the near field air-to-subsurface propagation formulas by utilizing the reciprocity theorem.

The four antennas considered — VED, VMD, HED, and HMD — are situated at a height h with respect to a cylindrical coordinate system (ρ, ϕ, z) and are assumed to carry a constant current I. The VED and HED are oriented in the z and x directions, respectively, and the axes of the VMD and HMD are oriented in the z and y directions, respectively. The various dipole orientations for the situation $h = 0^+$ are shown in Fig. 1. The plane, conducting, homogeneous earth occupies the lower half space ($z<0$) while the air occupies the upper half space ($z>0$). Meter-Kilogram-Second (MKS) units are employed and a time factor of $e^{i\omega t}$ is assumed.

---

[1] P.R. Bannister "Surface to Surface and Subsurface to Air Propagation - Quasi-Static and Near Field Ranges," paper presented at the AGARD/NATO symposium on Subsurface Communications, Paris, France, 25-29 April 1966; P.R. Bannister and W.C. Hart, The Near Fields of Subsurface Electric Dipole Antennas, USL Report No. 728, 4 March 1966; and P.R. Bannister and W.C. Hart, The Near Fields of Subsurface Magnetic Dipole Antennas, USL Report No. 729, 7 March 1966.

[2] K.A. Norton, "The Propagation of Radio Waves Over the Surface of the Earth and in the Upper Atmosphere," Proceedings of the I.R.E., vol. 25, no. 9, September 1937, pp. 1203-1236.

Table 1

SUBSURFACE-TO-AIR PROPAGATION FORMULAS FOR THE NEAR-FIELD RANGE

Dipole Type	E_ρ	E_ϕ	E_z
VED	$\frac{I d\ell\, e^{\gamma_1 h}}{2\pi\sigma_1} \frac{\rho e^{-\gamma_o R}}{R^5} \left[z\left(3 + 3\gamma_o R + \gamma_o^2 R^2\right) - \frac{\gamma_o^2}{\gamma_1} R^2 (1 + \gamma_o R) \right]$	0	$-\frac{I d\ell\, e^{\gamma_1 h}}{2\pi\sigma_1} \frac{e^{-\gamma_o R}}{R^3} \left[1 + \gamma_o R + \gamma_o^2 \rho^2 - \frac{3z^2}{R^2} (1 + \gamma_o R) \right]$
VMD	0	$-\frac{I dA\, e^{\gamma_1 h}}{2\pi\sigma_1} \frac{\rho e^{-\gamma_o R}}{R^5} \left[\left(3 + 3\gamma_1 z - \frac{15 z^2}{R^2}\right) (1 + \gamma_o R) + \left(1 + \gamma_1 z - \frac{6z^2}{R^2}\right) (\gamma_o R)^2 \right]$	0
HED	$\frac{I d\ell \cos\phi\, e^{\gamma_1 h}}{2\pi\sigma_1} \frac{e^{-\gamma_o R}}{R^3} \left[\left(1 - \gamma_1 z\right)(1 + \gamma_o R) + \gamma_o^2 R^2 \right]$	$\frac{I d\ell \sin\phi\, e^{\gamma_1 h}}{2\pi\sigma_1} \frac{e^{-\gamma_o R}}{R^3} \left[\left(2 + \gamma_1 z - \frac{3z^2}{R^2}\right) (1 + \gamma_o R) \right]$	$\frac{i\mu_o \omega I d\ell \cos\phi\, e^{\gamma_1 h}}{2\pi\gamma_1} \frac{\rho e^{-\gamma_o R}}{R^3} (1 + \gamma_o R)$
HMD	$\frac{i\mu_o \omega I dA \cos\phi\, e^{\gamma_1 h}}{2\pi\gamma_1} \frac{e^{-\gamma_o R}}{R^3} \left[\left(1 - \gamma_1 z\right)(1 + \gamma_o R) + \gamma_o^2 R^2 \right]$	$\frac{i\mu_o \omega I dA \sin\phi\, e^{\gamma_1 h}}{2\pi\gamma_1} \frac{e^{-\gamma_o R}}{R^3} \left[\left(2 + \gamma_1 z - \frac{3z^2}{R^2}\right) (1 + \gamma_o R) \right]$	$\frac{i\mu_o \omega I dA \cos\phi\, e^{\gamma_1 h}}{2\pi} \frac{e^{-\gamma_o R}}{R^3} \rho (1 + \gamma_o R)$

Table 1 (Cont'd)

SUBSURFACE-TO-AIR PROPAGATION FORMULAS FOR THE NEAR-FIELD RANGE

Dipole Type	H_ρ	H_ϕ	H_z
VED	0	$\frac{I d\ell}{2\pi}\frac{\gamma_0^2}{\gamma_1^2} e^{\gamma_1 h}\frac{\rho}{R^3}(1+\gamma_0 R)e^{-\gamma_0 R}$	0
VMD	$\frac{I dA\, e^{\gamma_1 h}}{2\pi\gamma_1}\frac{\rho e^{-\gamma_0 R}}{R^5}\left[\left(3-\frac{15z^2}{R^2}\right)(1+\gamma_0 R) + \left(1-\frac{6z^2}{R^2}\right)(\gamma_0 R)^2 - \frac{z^2}{R^2}(\gamma_0 R)^3\right]$	0	$-\frac{I dA\, e^{\gamma_1 h}}{2\pi\gamma_1^2}\frac{e^{-\gamma_0 R}}{R^5}\left\{\left[9(1+\gamma_1 z) - \frac{15z^2}{R^2}(6+\gamma_1 z) + \frac{105z^4}{R^4}\right](1+\gamma_0 R) + \left[4(1+\gamma_1 z) - \frac{6z^2}{R^2}\left(\frac{39}{6}+\gamma_1 z\right) + \frac{45z^4}{R^4}\right](\gamma_0 R)^2 + \left[(1+\gamma_1 z) - \frac{z^2}{R^2}(9+\gamma_1 z) + \frac{10z^4}{R^4}\right](\gamma_0 R)^3\right\}$
HED	$\frac{I d\ell \sin\phi\, e^{\gamma_1 h}}{2\pi\gamma_1}\frac{e^{-\gamma_0 R}}{R^3}\left[\left(2-\frac{3z^2}{R^2}\right)(1+\gamma_0 R) - \gamma_0^2 z^2\right]$	$-\frac{I d\ell \cos\phi\, e^{\gamma_1 h}}{2\pi\gamma_1}\frac{e^{-\gamma_0 R}}{R^3}\left[1+\gamma_0 R + \gamma_0^2 R^2\right]$	$\frac{I d\ell \sin\phi\, e^{\gamma_1 h}}{2\pi\gamma_1^2}\frac{e^{-\gamma_0 R}}{R^4}\left(\frac{\rho}{R}\right)\left[\left(3+3\gamma_1 z - \frac{15z^2}{R^2}\right)(1+\gamma_0 R) + \left(1+\gamma_1 z - \frac{6z^2}{R^2}\right)(\gamma_0 R)^2\right]$
HMD	$\frac{I dA \sin\phi\, e^{\gamma_1 h}}{2\pi}\frac{e^{-\gamma_0 R}}{R^3}\left\{\left(2-\frac{3z^2}{\rho^2}\right)(1+\gamma_0 R) - \gamma_0^2 z^2\right\}$	$-\frac{I dA \cos\phi\, e^{\gamma_1 h}}{2\pi}\frac{e^{-\gamma_0 R}}{R^3}\left(1+\gamma_0 R + \gamma_0^2 R^2\right)$	$\frac{I dA \sin\phi\, \rho e^{\gamma_1 h}}{2\pi\gamma_1}\frac{e^{-\gamma_0 R}}{R^5}\left[(3+3\gamma_1 z - 15z^2/R^2)(1+\gamma_0 R) + (1+\gamma_1 z - 6z^2/R^2)(\gamma_0 R)^2\right]$

Table 2

SUBSURFACE-TO-AIR PROPAGATION FORMULAS FOR THE NEAR-FIELD RANGE WITH $\rho >> z$

Dipole Type	E_ρ	E_ϕ	E_z
VED	$-\frac{I\,d\ell}{2\pi\sigma_1}\frac{e^{\gamma_1 h}}{\rho^4}\left[\left(\frac{\gamma_o^2}{\gamma_1}\right)(1+\gamma_o\rho)\rho^2 - z(3+3\gamma_o\rho+\gamma_o^2\rho^2)\right]e^{-\gamma_o\rho}$	0	$-\frac{I\,d\ell}{2\pi\sigma_1}e^{\gamma_1 h}\frac{e^{-\gamma_o\rho}}{\rho^3}(1+\gamma_o\rho+\gamma_o^2\rho^2)$
VMD	0	$-\frac{I\,dA\,e^{\gamma_1 h}}{2\pi\sigma_1}\frac{e^{-\gamma_o\rho}}{\rho^4}$ $(3+3\gamma_o\rho+\gamma_o^2\rho^2)(1+\gamma_1 z)$	0
HED	$\frac{I\,d\ell\cos\phi\,e^{\gamma_1 h}}{2\pi\sigma_1}\frac{e^{-\gamma_o\rho}}{\rho^3}$ $[(1-\gamma_1 z)(1+\gamma_o\rho)+\gamma_o^2\rho^2]$	$\frac{I\,d\ell\sin\phi\,e^{\gamma_1 h}}{2\pi\sigma_1}\frac{e^{-\gamma_o\rho}}{\rho^3}$ $[(2+\gamma_1 z)(1+\gamma_o\rho)]$	$\frac{i\mu_o\omega I\,d\ell\cos\phi\,e^{\gamma_1 h}}{2\pi\gamma_1}\frac{e^{-\gamma_o\rho}}{\rho^2}[1+\gamma_o\rho]$
HMD	$\frac{i\mu_o\omega I\,dA\cos\phi\,e^{\gamma_1 h}}{2\pi\gamma_1}\frac{e^{-\gamma_o\rho}}{\rho^3}$ $[(1-\gamma_1 z)(1+\gamma_o\rho)+\gamma_o^2\rho^2]$	$\frac{i\mu_o\omega I\,dA\sin\phi\,e^{\gamma_1 h}}{2\pi\gamma_1}\frac{e^{-\gamma_o\rho}}{\rho^3}$ $[(2+\gamma_1 z)(1+\gamma_o\rho)]$	$\frac{i\mu_o\omega I\,dA\cos\phi\,e^{\gamma_1 h}}{2\pi}\frac{e^{-\gamma_o\rho}}{\rho^2}(1+\gamma_o\rho)$

Table 2 (Cont'd)

SUBSURFACE-TO-AIR PROPAGATION FORMULAS FOR THE NEAR-FIELD RANGE WITH $\rho >> z$

Dipole Type	H_ρ	H_ϕ	H_z
VED	0	$\frac{I d\ell}{2\pi\rho^2}\frac{\gamma_o^2}{\gamma_1^2}(1+\gamma_o\rho)\,e^{\gamma_1 h}\,e^{-\gamma_o\rho}$	0
VMD	$-\frac{I dA\, e^{\gamma_1 h}}{2\pi\gamma_1}\frac{e^{-\gamma_o\rho}}{\rho^4}(3+3\gamma_o\rho+\gamma_o^2\rho^2)$	0	$-\frac{I dA\, e^{\gamma_1 h}}{2\pi\gamma_1^2}\frac{e^{-\gamma_o\rho}}{\rho^5}$ $(9+9\gamma_o\rho+4\gamma_o^2\rho^2+\gamma_o^3\rho^3)(1+\gamma_1 z)$
HED	$\frac{I d\ell\,\sin\phi\, e^{\gamma_1 h}}{\pi\gamma_1}\ \frac{e^{-\gamma_o\rho}}{\rho^3}\ [1+\gamma_o\rho]$	$-\frac{I d\ell\,\cos\phi\, e^{\gamma_1 h}}{2\pi\gamma_1}\frac{e^{-\gamma_o\rho}}{\rho^3}$ $[1+\gamma_o\rho+\gamma_o^2\rho^2]$	$\frac{I d\ell\,\sin\phi\, e^{\gamma_1 h}}{2\pi\gamma_1^2}\ \frac{e^{-\gamma_o\rho}}{\rho^4}$ $[(1+\gamma_1 z)(3+3\gamma_o\rho+\gamma_o^2\rho^2)]$
HMD	$\frac{I dA\,\sin\phi\, e^{\gamma_1 h}}{2\pi}\frac{e^{-\gamma_o\rho}}{\rho^3}[2(1+\gamma_o\rho)]$	$-\frac{I dA\,\cos\phi\, e^{\gamma_1 h}}{2\pi}\ \frac{e^{-\gamma_o\rho}}{\rho^3}$ $[1+\gamma_o\rho+\gamma_o^2\rho^2]$	$\frac{I dA\,\sin\phi\, e^{\gamma_1 h}}{2\pi\gamma_1}\ \frac{e^{-\gamma_o\rho}}{\rho^4}$ $[(3+3\gamma_o\rho+\gamma_o^2\rho^2)(1+\gamma_1 z)]$

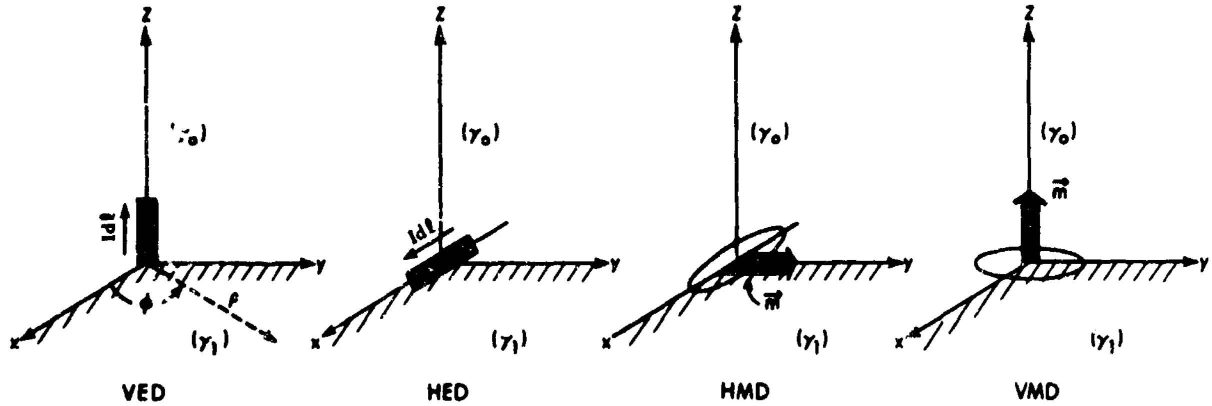

Fig. 1 - Dipole Orientations when $h = 0^+$

RECIPROCITY THEOREM

The reciprocity theorem (applicable to dipoles in the presence of any linear media) states that the voltage V_2 induced in antenna 2 by current I_1 of antenna 1 is the same as the voltage V_1 induced in antenna 1 by an identical current I_2 flowing in antenna 2. (For further details, see Carson,[3] Galejs,[4] or Wait.[5]) Application of this theorem, which utilizes the geometry in Fig. 1, results in

$$E_z^{HM}[z, h] = i\,\omega\,\mu_0\, H_\phi^{VE} \cos\phi\, \frac{dA}{dl}\,[h, z]\,, \tag{1}$$

where

$[a, b]$ = [height or depth of transmitting dipole, height or depth of receiving dipole] ;

---

[3] J.R. Carson, "Reciprocal Theorems in Radio Communication," Proceedings of the I.R.E., vol. 17, no. 6, June 1929, pp. 952-956.

[4] J. Galejs, "Excitation of VLF and ELF Radio Waves by a Horizontal Magnetic Dipole," Radio Science, Journal of Research, National Bureau of Standards, vol. 65D, no. 3 May-June 1961, pp. 305-311.

[5] J.R. Wait, Electromagnetic Waves in Stratified Media, Pergamon Press, Oxford, 1962, pp. 168 - 174.

$$H_z^{HM}[z,h] = -H_\rho^{VM}\sin\phi\,\frac{dA^{HM}}{dA^{VM}}[h,z]\,; \tag{2}$$

$$E_z^{HE}[z,h] = -E_\rho^{VE}\cos\phi\,\frac{dl^{HE}}{dl^{VE}}[h,z]\,; \tag{3}$$

$$H_z^{HE}[z,h] = \frac{-1}{i\,\omega\,\mu_0}\,E_\phi^{VM}\sin\phi\,\frac{dl}{dA}[h,z]\,; \tag{4}$$

$$H_\rho^{HE}[z,h] = \frac{1}{i\,\omega\,\mu_0}\,E_\phi^{HM}\,\frac{dl}{dA}[h,z]\,; \tag{5}$$

$$H_\phi^{HE}[z,h] = \frac{-1}{i\,\omega\,\mu_0}\,E_\rho^{HM}\,\frac{dl}{dA}[h,z]\,; \tag{6}$$

and

$$E_z^{VE}, H_z^{VM}, H_\rho^{HM}, H_\phi^{HM}, E_\rho^{HE}, E_\phi^{HE}[z,h] = E_z^{VE}, H_z^{VM}, H_\rho^{HM}, H_\phi^{HM}, E_\rho^{HE}, E_\phi^{HE}[h,z]\,. \tag{7}$$

DERIVATION OF THE NEAR FIELD ELECTRIC AND MAGNETIC FIELD COMPONENTS

By employing the results listed in Tables 1 and 2 and Eqs. (1) through (7), the near field air-to-subsurface field component expressions, which are valid for $|\gamma_1 R'| >> 1$ (i.e., $R' >> \delta$), $h \geq 0^+$, $z \leq 0^-$, and $R' >> |z|$, can be determined. The additional restriction ($|\gamma_0^2\rho/\gamma_1| << 1$) is required for the vertically polarized components. Also, ionospheric effects are neglected. These field component expressions are as follows:

for the vertical electric dipole,

$$E_\rho \cong \frac{-I\,dl\,i\,\mu_0\,\omega\,e^{\gamma_1 z}}{2\pi\gamma_1}\;\frac{\rho\,e^{-\gamma_0 R'}}{(R')^3}\,(1+\gamma_0 R')\,, \tag{8}$$

$$E_z \cong \frac{-I\,dl\,e^{\gamma_1 z}}{2\pi\sigma_1}\;\frac{e^{-\gamma_0 R'}}{(R')^3}\left[1+\gamma_0 R'+\gamma_0^2\rho^2-\frac{3h^2}{(R')^2}(1+\gamma_0 R')\right], \tag{9}$$

and

$$H_\phi \cong \frac{I\,dl\,e^{\gamma_1 z}}{2\pi}\;\frac{\rho\,e^{-\gamma_0 R'}}{(R')^3}\,(1+\gamma_0 R')\,; \tag{10}$$

for the vertical magnetic dipole,

$$\begin{aligned} E_\phi \cong \frac{-I\,dA\,e^{\gamma_1 z}}{2\pi\sigma_1}\;\frac{e^{-\gamma_0 R'}}{(R')^4}\left(\frac{\rho}{R'}\right)\Bigg[&\left(3+3\gamma_1 h-\frac{15h^2}{(R')^2}\right)(1+\gamma_0 R') \\ &+\left(1+\gamma_1 h-\frac{6h^2}{(R')^2}\right)(\gamma_0 R')^2\Bigg]\,, \end{aligned} \tag{11}$$

$$\begin{aligned} H_\rho \cong \frac{-I\,dA\,e^{\gamma_1 z}}{2\pi\gamma_1}\;\frac{\rho\,e^{-\gamma_0 R'}}{(R')^5}\Bigg[&\left(3+3\gamma_1 h-\frac{15h^2}{(R')^2}\right)(1+\gamma_0 R') \\ &+\left(1+\gamma_1 h-\frac{6h^2}{(R')^2}\right)(\gamma_0 R')^2\Bigg]\,, \end{aligned} \tag{12}$$

and

$$H_z \cong \frac{-I\,d\,A\,e^{\gamma_1 z}}{2\pi\gamma_1^2}\;\frac{e^{-\gamma_0 R'}}{(R')^5}\Bigg\{\Bigg[9\,(1+\gamma_1 h) - \frac{15\,h^2}{(R')^2}(6+\gamma_1 h)$$

$$+\frac{105\,h^4}{(R')^4}\Bigg]\Big[1+\gamma_0 R'\Big] + \Bigg[4\,(1+\gamma_1 h) - \frac{6\,h^2}{(R')^2}\left(\frac{39}{6}+\gamma_1 h\right) \qquad (13)$$

$$+\frac{45\,h^4}{(R')^4}\Bigg](\gamma_0 R')^2 + \Bigg[1+\gamma_1 h - \frac{h^2}{(R')^2}(9+\gamma_1 h) + \frac{10\,h^4}{(R')^4}\Bigg](\gamma_0 R')^3\Bigg\};$$

for the horizontal electric dipole,

$$E_\rho \cong \frac{I\,d\,l\cos\phi\,e^{\gamma_1 z}}{2\pi\sigma_1}\;\frac{e^{-\gamma_0 R'}}{(R')^3}\Big[(1-\gamma_1 h)\,(1+\gamma_0 R') + (\gamma_0 R')^2\Big], \qquad (14)$$

$$E_\phi \cong \frac{I\,d\,l\sin\phi\,e^{\gamma_1 z}}{2\pi\sigma_1}\;\frac{e^{-\gamma_0 R'}}{(R')^3}\Bigg[\left(2+\gamma_1 h - \frac{3\,h^2}{(R')^2}\right)(1+\gamma_0 R')\Bigg], \qquad (15)$$

$$E_z \cong \frac{-I\,d\,l\cos\phi\,e^{\gamma_1 z}}{2\pi\sigma_1}\;\frac{\rho\,e^{-\gamma_0 R'}}{(R')^5}\Bigg[h\left(3+3\,\gamma_0 R' + (\gamma_0 R')^2\right) \qquad (16)$$

$$-\frac{\gamma_0^2\,(R')^2}{\gamma_1}(1+\gamma_0 R')\Bigg],$$

$$H_\rho \cong \frac{I\,d\,l\sin\phi\,e^{\gamma_1 z}}{2\pi\gamma_1}\;\frac{e^{-\gamma_0 R'}}{(R')^3}\Bigg[\left(2+\gamma_1 h - \frac{3\,h^2}{(R')^2}\right)(1+\gamma_0 R')\Bigg] \qquad (17)$$

$$H_\phi \cong \frac{-I\,d\,l\cos\phi\,e^{\gamma_1 z}}{2\pi\gamma_1}\;\frac{e^{-\gamma_0 R'}}{(R')^3}\Big[(1-\gamma_1 h)\,(1+\gamma_0 R') + (\gamma_0 R')^2\Big], \qquad (18)$$

and

$$H_z \cong \frac{I\,d\,l \sin\phi\, e^{\gamma_1 z}}{2\pi\gamma_1^2} \frac{\rho\, e^{-\gamma_0 R'}}{(R')^5} \left[\left(3 + 3\gamma_1 h - \frac{15 h^2}{(R')^2}\right)(1 + \gamma_0 R') + \left(1 + \gamma_1 h - \frac{6 h^2}{(R')^2}\right)(\gamma_0 R')^2\right] ; \tag{19}$$

and for the horizontal magnetic dipole,

$$E_\rho \cong \frac{I\,d\,A\, i\,\mu_0\,\omega \cos\phi\, e^{\gamma_1 z}}{2\pi\gamma_1} \frac{e^{-\gamma_0 R'}}{(R')^3} \left[1 + \gamma_0 R' + (\gamma_0 R')^2\right] , \tag{20}$$

$$E_\phi \cong \frac{I\,d\,A\, i\,\mu_0\,\omega \sin\phi\, e^{\gamma_1 z}}{2\pi\gamma_1} \frac{e^{-\gamma_0 R'}}{(R')^3} \left[\left(2 - \frac{3 h^2}{(R')^2}\right)(1 + \gamma_0 R') - \gamma_0^2 h^2\right] , \tag{21}$$

$$E_z \cong \frac{I\,d\,A\, i\,\mu_0\,\omega \cos\phi\, e^{\gamma_1 z}}{2\pi\gamma_1} \left(\frac{\gamma_0^2 \rho}{\gamma_1}\right) \frac{e^{-\gamma_0 R'}}{(R')^3} (1 + \gamma_0 R') , \tag{22}$$

$$H_\rho \cong \frac{I\,d\,A \sin\phi\, e^{\gamma_1 z}}{2\pi} \frac{e^{-\gamma_0 R'}}{(R')^3} \left[\left(2 - \frac{3 h^2}{(R')^2}\right)(1 + \gamma_0 R') - \gamma_0^2 h^2\right] , \tag{23}$$

$$H_\phi \cong \frac{-I\,d\,A \cos\phi\, e^{\gamma_1 z}}{2\pi} \frac{e^{-\gamma_0 R'}}{(R')^3} \left[1 + \gamma_0 R' + (\gamma_0 R')^2\right] , \tag{24}$$

and

$$H_z \cong \frac{I\,d\,A \sin\phi\, e^{\gamma_1 z}}{2\pi\gamma_1} \frac{\rho\, e^{-\gamma_0 R'}}{(R')^5} \left[\left(3 - \frac{15 h^2}{(R')^2}\right)(1 + \gamma_0 R') + \left(1 - \frac{6 h^2}{(R')^2}\right)(\gamma_0 R')^2 - \frac{h^2}{(R')^2}(\gamma_0 R')^3\right] . \tag{25}$$

When $|\gamma_0 R'| << 1$, all these expressions reduce to the quasi-near range results derived by Bannister.[6] Furthermore, when $\rho >> h$, these expressions reduce to the following well-known results:

for the vertical electric dipole when $\rho >> h$,

$$E_\rho \cong \frac{-i \mu_0 \omega I\, dl\, e^{\gamma_1 z}}{2 \pi \gamma_1} \frac{e^{-\gamma_0 \rho}}{\rho^2} (1 + \gamma_0 \rho), \tag{26}$$

$$E_z \cong \frac{-I\, dl\, e^{\gamma_1 z}}{2 \pi \sigma_1} \frac{e^{-\gamma_0 \rho}}{\rho^3} (1 + \gamma_0 \rho + \gamma_0^2 \rho^2), \tag{27}$$

and

$$H_\phi \cong \frac{I\, dl\, e^{\gamma_1 z}}{2 \pi} \frac{e^{-\gamma_0 \rho}}{\rho^2} (1 + \gamma_0 \rho); \tag{28}$$

for the vertical magnetic dipole when $\rho >> h$,

$$E_\phi \cong \frac{-I\, dA\, e^{\gamma_1 z}}{2 \pi \sigma_1} \frac{e^{-\gamma_0 \rho}}{\rho^4} [(1 + \gamma_1 h)(3 + 3 \gamma_0 \rho + \gamma_0^2 \rho^2)], \tag{29}$$

$$H_\rho \cong \frac{-I\, dA\, e^{\gamma_1 z}}{2 \pi \gamma_1} \frac{e^{-\gamma_0 \rho}}{\rho^4} [(1 + \gamma_1 h)(3 + 3 \gamma_0 \rho + \gamma_0^2 \rho^2)], \tag{30}$$

and

$$H_z \cong \frac{-I\, dA\, e^{\gamma_1 z}}{2 \pi \gamma_1^2} \frac{e^{-\gamma_0 \rho}}{\rho^5} [(1 + \gamma_1 h)(9 + 9 \gamma_0 \rho + 4 \gamma_0^2 \rho^2 + \gamma_0^3 \rho^3)]; \tag{31}$$

---

[6] P.R. Bannister, "The Quasi-Near Fields of Dipole Antennas," a paper being prepared for submission to the IEEE PGAP.

for the horizontal electric dipole when $\rho >> h$,

$$E_\rho \cong \frac{I\,dl \cos\phi\, e^{\gamma_1 z}}{2\pi\sigma_1} \frac{e^{-\gamma_0 \rho}}{\rho^3} [(1-\gamma_1 h)(1+\gamma_0 \rho)+\gamma_0^2 \rho^2], \tag{32}$$

$$E_\phi \cong \frac{I\,dl \sin\phi\, e^{\gamma_1 z}}{2\pi\sigma_1} \frac{e^{-\gamma_0 \rho}}{\rho^3} [(2+\gamma_1 h)(1+\gamma_0 \rho)], \tag{33}$$

$$E_z \cong \frac{-I\,dl \cos\phi\, e^{\gamma_1 z}}{2\pi\sigma_1} \frac{e^{-\gamma_0 \rho}}{\rho^4} \left[h\,(3+3\,\gamma_0 \rho+\gamma_0^2 \rho^2)-\frac{\gamma_0^2 \rho^2}{\gamma_1}(1+\gamma_0 \rho)\right], \tag{34}$$

$$H_\rho \cong \frac{I\,dl \sin\phi\, e^{\gamma_1 z}}{2\pi\gamma_1} \frac{e^{-\gamma_0 \rho}}{\rho^3} [(2+\gamma_1 h)(1+\gamma_0 \rho)], \tag{35}$$

$$H_\phi \cong \frac{-I\,dl \cos\phi\, e^{\gamma_1 z}}{2\pi\gamma_1} \frac{e^{-\gamma_0 \rho}}{\rho^3} [(1-\gamma_1 h)(1+\gamma_0 \rho)+\gamma_0^2 \rho^2], \tag{36}$$

and

$$H_z \cong \frac{I\,dl \sin\phi\, e^{\gamma_1 z}}{2\pi\gamma_1^2} \frac{e^{-\gamma_0 \rho}}{\rho^4} [(1+\gamma_1 h)(3+3\,\gamma_0 \rho+\gamma_0^2 \rho^2)]; \tag{37}$$

and for the horizontal magnetic dipole when $\rho >> h$,

$$E_\rho \cong \frac{I\,dA\, i\,\mu_0\,\omega \cos\phi\, e^{\gamma_1 z}}{2\pi\gamma_1} \frac{e^{-\gamma_0 \rho}}{\rho^3} (1+\gamma_0 \rho+\gamma_0^2 \rho^2), \tag{38}$$

$$E_\phi \cong \frac{I\,dA\, i\,\mu_0\,\omega \sin\phi\, e^{\gamma_1 z}}{\pi\gamma_1} \frac{e^{-\gamma_0 \rho}}{\rho^3} (1+\gamma_0 \rho), \tag{39}$$

$$E_z \cong \frac{I\,dA\, i\,\mu_0\,\omega \cos\phi\, e^{\gamma_1 z}}{2\pi\gamma_1} \left(\frac{\gamma_0^2}{\gamma_1}\right) \frac{e^{-\gamma_0 \rho}}{\rho^2} (1+\gamma_0 \rho), \tag{40}$$

$$H_\rho \cong \frac{I\, d\, A \sin\phi\, e^{\gamma_1 z}}{\pi} \; \frac{e^{-\gamma_0 \rho}}{\rho^3} (1 + \gamma_0 \rho) , \tag{41}$$

$$H_\phi \cong \frac{-I\, d\, A \cos\phi\, e^{\gamma_1 z}}{2\pi} \; \frac{e^{-\gamma_0 \rho}}{\rho^3} (1 + \gamma_0 \rho + \gamma_0^2 \rho^2) , \tag{42}$$

and

$$H_z \cong \frac{I\, d\, A \sin\phi\, e^{\gamma_1 z}}{2\pi\gamma_1} \; \frac{e^{-\gamma_0 \rho}}{\rho^4} (3 + 3\gamma_0 \rho + \gamma_0^2 \rho^2) . \tag{43}$$

When $\rho >> h$, only the VMD and HED field component expressions have transmitting antenna height gain factors. The height gain factor is defined as the ratio of the field strength (at some depth z) when the transmitting antenna is at a height h to that when the transmitting antenna is a height $h = 0^+$. Some numerical results for the various height gain factors when $|\gamma_0 \rho| << 1$ are given by Bannister and Hart.[7] (Note that z must be replaced by h and Eqs. (1) through (7) must be employed in order to apply the results obtained by Bannister and Hart to the air-to-subsurface propagation case.)

CONCLUSIONS

The air-to-subsurface electric and magnetic field components produced by vertical and horizontal dipoles located above the surface of a plane, conducting, homogeneous earth have been derived for the near field range. Ionospheric effects have been neglected. When $\rho >> h$, these field component expressions reduce to well-known expressions. In addition, when $|\gamma_0 R'| << 1$, they reduce to the quasi-near range formulas.

---

[7] P.R. Bannister and W.C. Hart, The Quasi-Static Fields of Dipole Antennas - Part II, USL Report No. 719, 8 February 1966; Part III, USL Report No. 720, 23 February 1966.

UNCLASIFIED
Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) U. S. Navy Underwater Sound Laboratory Fort Trumbull, New London, Connecticut	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b GROUP

3. REPORT TITLE

UTILIZATION OF THE RECIPROCITY THEOREM TO DETERMINE THE NEAR FIELD AIR-TO-SUBSURFACE PROPAGATION FORMULAS

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Research Report

5. AUTHOR(S) (Last name, first name, initial)

Bannister, Peter R.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
22 November 1966	18	10

8a. CONTRACT OR GRANT NO. b. PROJECT NO. SF 106 01 02 Task 7077 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 786 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. AVAILABILITY/LIMITATION NOTICES

Distribution of this document is unlimited

11. SUPPLEMENTARY NOTES	12 SPONSORING MILITARY ACTIVITY
	U. S. Navy

13. ABSTRACT

The electric and magnetic field components produced by vertical and horizontal dipoles (both electric and magnetic types) located at or above the surface of a plane, conducting, homogeneous earth are derived for the near field range. The height h of the transmitting antenna is $\geq 0^+$, while the depth z of the receiving antenna is $\leq 0^-$ (air-to-subsurface propagation). Ionospheric effects are neglected

The derivations are based upon application of the reciprocity theorem to previously derived field components. It is observed that these equations reduce to well-known expressions when the horizontal separation (ρ) between the transmitting and receiving antennas is much greater than h.

DD FORM 1 JAN 64 1473

UNCLASSIFIED
Security Classification

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Vertical Electric Dipole						
Vertical Magnetic Dipole						
Horizontal Electric Dipole						
Horizontal Magnetic Dipole						
Antenna Electric Fields						
Antenna Magnetic Fields						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2a. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7a. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8a. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through
_______________________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through
_______________________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through
_______________________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.

UNCLASSIFIED
Security Classification